Math 270 Day 8

Chapter 3: Applications of Linear Differential Equations

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What we'll go over in this lecture

- Section 3.2: Radioactive Decay
- Section 3.3: Newton's Law of Cooling
- Section 3.4: Newtonian Mechanics

Section 3.2: Radioactive Decay

Radioactive Decay

t = time
$x(t)$ = amount (mass or weight) of radioactive material remaining at time t

IVP: $\frac{dx}{dt} = kx$, $x(0) = x_0$, where k is a (negative) constant (discuss)

Solution: $x(t) = x_0 e^{kt}$

Section 3.2: Radioactive Decay

Radioactive Decay

Ex 1: A radioactive substance has a half-life of 7 years. If initially there are 300g of a radioactive substance,…

a) Find a formula for the amount of the radioactive substance remaining after t years

Section 3.2: Radioactive Decay

Radioactive Decay

Ex 1: A radioactive substance has a half-life of 7 years. If initially there are 300g of a radioactive substance,…

b) How much of the radioactive substance will remain after 12 years?

Section 3.2: Radioactive Decay

Radioactive Decay

Ex 1: A radioactive substance has a half-life of 7 years. If initially there are 300g of a radioactive substance,…

c) When will only 10g of the radioactive substance remain?

Section 3.3: Newton's Law of Cooling

- Newton's Law of Cooling is used to determine the temperature of an object T as a function of time t when the object is placed in a reservoir

- Newton's Law of Cooling states that the current rate of change of the temperature of the object is directly proportional to the current temperature difference between the object and the reservoir

$$\text{IVP:} \quad \frac{dT}{dt} = k(T_R - T) \ , \ T(0) = T_0$$

t = time
$T(t)$ = temperature of the object at time t
T_R = temp of reservoir (usually constant)
k = a positive constant

Section 3.3: Newton's Law of Cooling

Ex 2: A $130^{\circ}F$ cup of coffee is put in a freezer that is set at $-5^{\circ}F$. 10 minutes later the temperature of the coffee is measured to be $95^{\circ}F$.

Let T be the temperature of the coffee in degrees Fahrenheit
t minutes after the coffee was first put into the freezer.

a) What is the IVT for this problem?

Section 3.3: Newton's Law of Cooling

Ex 2: A $130^{\circ}F$ cup of coffee is put in a freezer that is set at $-5^{\circ}F$. 10 minutes later the temperature of the coffee is measured to be $95^{\circ}F$.

Let T be the temperature of the coffee in degrees Fahrenheit
t minutes after the coffee was first put into the freezer.

b) Find a formula for the temperature T of the coffee as a function of time t.

Section 3.3: Newton's Law of Cooling

Ex 2: A $130^{\circ}F$ cup of coffee is put in a freezer that is set at $-5^{\circ}F$. 10 minutes later the temperature of the coffee is measured to be $95^{\circ}F$.

Let T be the temperature of the coffee in degrees Fahrenheit
t minutes after the coffee was first put into the freezer.

c) What will the temperature of the coffee be an hour after it was first put into the freezer?

Section 3.3: Newton's Law of Cooling

Ex 2: A $130^\circ F$ cup of coffee is put in a freezer that is set at $-5^\circ F$. 10 minutes later the temperature of the coffee is measured to be $95^\circ F$.

Let T be the temperature of the coffee in degrees Fahrenheit
t minutes after the coffee was first put into the freezer.

d) How long will it take the coffee to reach $72^\circ F$?

Section 3.3: Newton's Law of Cooling

Ex 2: A $130^{\circ}F$ cup of coffee is put in a freezer that is set at $-5^{\circ}F$. 10 minutes later the temperature of the coffee is measured to be $95^{\circ}F$.

Let T be the temperature of the coffee in degrees Fahrenheit
t minutes after the coffee was first put into the freezer.

e) What is the limiting temperature of the coffee?

Section 3.4: Newtonian Mechanics

- In this section, we are only going to focus on an object falling straight downward with only 2 forces on it (gravity and air resistance)

Discuss Some Types of Forces and Their Dependence on t , x , and v
Hand, Spring, Gravitational, Air Resistance

Section 3.4: Newtonian Mechanics

- In this section, we are only going to focus on an object falling straight downward with only 2 forces on it (gravity and air resistance)

Discuss Newton's Second Law

$F(t, x, v) = ma \rightarrow F(t, v) = m\frac{dv}{dt}$

Section 3.4: Newtonian Mechanics

Ex 3 (sec. 3.4 book ex 2 modified): An object of mass 3 kg is released from rest 500 m above the ground and allowed to fall under the influence of gravity. Assume the gravitational force is constant, with g = 9.81 m/s^2, and the force due to air resistance is proportional to the velocity of the object with proportionality constant $b = 3\ N \cdot sec/m$.

a) What is the IVP for this problem?

Section 3.4: Newtonian Mechanics

Ex 3 (sec. 3.4 book ex 2 modified): An object of mass 3 kg is released from rest 500 m above the ground and allowed to fall under the influence of gravity. Assume the gravitational force is constant, with g = 9.81 m/s^2, and the force due to air resistance is proportional to the velocity of the object with proportionality constant $b = 3\ N \cdot sec/m$.

b) Find a formula for the position of the object at time t

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c) When will the object strike the ground? What will the object's velocity be at that time?

Section 3.4: Newtonian Mechanics

Ex 3 (sec. 3.4 book ex 2 modified): An object of mass 3 kg is released from rest 500 m above the ground and allowed to fall under the influence of gravity. Assume the gravitational force is constant, with g = 9.81 m/s^2, and the force due to air resistance is proportional to the velocity of the object with proportionality constant $b = 3\ N \cdot sec/m$.

d) What is the limiting velocity of the object (if it could fall forever without ever reaching the ground)

Section 3.4: Newtonian Mechanics

Ex 4 (sec. 3.4 book ex 3 modified): A parachutist whose mass is 75 kg drops from a helicopter hovering 4000 m above the ground and falls towards the earth under the influence of gravity. Assume the gravitational force is constant. Assume also that the force due to air resistance is proportional to the velocity of the parachutist, with proportionality constant $b_1 = 15\ N \cdot sec/m$ when the chute is closed and with constant $b_2 = 105\ N \cdot sec/m$ when the chute is open. If the chute does not open until 1 min after the parachutist leaves the helicopter, after how many seconds will she reach the ground?